Spatial nonparaxial correction of the ultrashort pulsed beam propagation in free space

Xiquan Fu, Hong Guo,* Wei Hu, and Song Yu
Laboratory of Light Transmission Optics, South China Normal University, Guangzhou 510631, People's Republic of China

(Received 14 October 2001; published 14 May 2002)

In this paper, a family of integral solutions representing ultrashort pulsed beam propagation in free space is studied by using the comoving frame coordinates and the Fourier transformation for time variable in terms of well-known paraxial approximation. The pulsed Gaussian-like beam solution is obtained as a special case of the integral solution, where the pulsed Gaussian beam solution is included. Further, starting from the non-paraxial pulsed beam propagation equation in the temporal-frequency domain and making use of the spatial Fourier transform, the nonparaxial pulsed beam solution is derived based on the paraxial pulsed beam solution, where the nonparaxiality is evaluated by a series of expansions.

DOI: 10.1103/PhysRevE.65.056611 PACS number(s): 42.25.Bs, 42.65.Re, 41.20.Jb

I. INTRODUCTION

The rapid advancement in laser technology in the past few decades has brought about the production of extremely short laser pulses, containing only a few, even only one, cycles of optical oscillations, which led to many new questions. For long laser pulse propagation, it is well known that the scalar and paraxial approximation of diffraction theory provides a successful description of the propagation of space modulated light waves without considering their temporal structure [1–3]. For extremely short laser pulse propagation, however, the diffraction of light waves with spatial and temporal modulation has to be considered simultaneously [4]. Several authors have theoretically discussed the propagation of the ultrashort pulses using a series of techniques, such as the scalar approximation, vector analysis, paraxial approximation, slowly varying envelop approximations (SVEA), and complex analytical signal (CAS) theory.

One of the key features of the ultrashort pulsed beam solutions obtained under the scalar and paraxial approximation is that the spatiotemporal coupling leads to substantial pulsed beam reshaping through diffraction even when such pulses propagate in free space [5–12]. Owing to the mathematical simplicity of the Gaussian function, most of the research was carried out based on the spatial Gaussian distribution. This is a family of solutions called pulsed Gaussian beam (PGB), in which the spatiotemporal coupling occurs [10–13]. On the other hand, a method based on the CAS representation of polychromatic light [1] has been applied to analytically obtain a physical solution for an ultrashort pulsed beam, which overcomes the spatial singularity due to SVEA when the pulse is extremely short [13]. The difference between the CAS method and SVEA has been analyzed previously [10–12], and the propagation of half-cycle electromagnetic pulses centered at terahertz frequencies in free space has been studied [5]. It is shown that [5] the temporal pulse shape of an aperture half-cycle pulse retains much of its unipolar character after traveling more than 20 times the aperture dimensions, although it is significantly altered during propagation.

Many other new phenomena have been observed while some theoretical conclusions have been drawn, among which the Gouy phase shift resulting in a pulse-to-pulse temporal instability [14–18], the far-field propagation demonstrating common patterns of time-derivative behavior regardless of the initial spatiotemporal profiles [6,7], and linear homogeneous nondispersive and dispersive medium have become a subject of interest [19,20]. Starting from Maxwell's equations, exact solutions have been obtained [14–18], to describe the evolution of single-cycle pulsed beam, which are partially discovered by Ziolkowski [21] from the free-space wave equation and by using the method of Hertz potentials. These solutions demonstrated that (i) the Gouy phase shift of focused beams leads to temporal reshaping and polarity reversals of single-cycle terahertz pulses; (ii) the real and imaginary parts of the pulse beam solutions are related by Hilbert transform, which coincides with the CAS solutions [1]; (iii) these paraxial solutions are the natural spatiotemporal modes of an open electromagnetic cavity. The different techniques have been applied, such as the well-known ABCD matrices of Gaussian beam optics [17], and the simulation. A family of space-time nonseparable analytic solutions describing spatiotemporal dynamics of isodiffracting single-cycle and few-cycle pulses with Hermite-Gaussian and Laguerre-Gaussian transverse profiles have been presented, and the creation of "dark pulses" at certain transverse positions has been analyzed [17,22]. The properties of single-cycle terahertz pulses propagating through a focus were investigated experimentally (on-axis case) and numerically [18] and the pulse distortions of a focused single-cycle pulse were illustrated. In terms of the Gouy shift, the changes in pulse shape from antisymmetric to symmetric can be understood, while major pulse distortions arise from diffraction effects that can be precisely modeled by numerical solution of the time-domain diffraction integral [18].

It is more difficult and complicated to derive the solutions of the ultrashort pulsed beam by CAS than by SVEA, whereas the solutions of SVEA are inconsistent with physical significance. It has been pointed out [23] that the diffraction of few-cycle light pulses can be solved by means of a perturbation technique based on the SVEA solution and the

*Author to whom correspondence should be addressed. FAX: +86-20-8521-1603. Email address: hguo@scnu.edu.cn

propagated field is expressed as a series of correction terms to the field obtained from diffraction laws for many-cycle pulses. Mathematically, this method is similar to that proposed by Lax *et al.* [24] and the above work is based on the paraxial approximation. In this paper, we will consider nonparaxial corrections to the paraxial solutions of an ultrashort pulsed beam. This can be conducted by using a small dimensionless parameter $1/kw_0$ (here k is the wave number in free space and w_0 is the waist width of the beam) for correction [24], together with the use of a truncation operator to correct the paraxial beam solutions [25,26]. A more general paraxial integral solution of an ultrashort pulsed beam will be derived, by which we will give the pulsed Gaussian-like beam solution, in which PGB is included. Then, the nonparaxial solutions can be derived by using the Fourier transform technique, which is a correction to the integral solution of the paraxial ultrashort pulsed beam.

The paper is organized as follows. In Sec. II, the ultrashort pulsed beam propagation under paraxial approximation is reviewed, where the integral solution is obtained by making use of the Fourier transform for the reduced time variable $\tau = t - z/c$. Then we derive a family of solutions of pulsed Gaussian-like beam, in which the PGB is included. In Sec. III, by applying the Fourier transform for spatial coordinates $\vec{r}$ and the nonparaxial wave equation in the temporal-frequency domain, the nonparaxial correction to the paraxial pulsed beam solutions can be derived as a series of expansions. The paper concludes with a discussion in Sec. IV.

II. PARAXIAL PROPAGATION OF AN ULTRASHORT PULSED BEAM

The free-space propagation of an electromagnetic pulse is governed by the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E(\vec{r},z,t) = 0, \tag{1}$$

where $\vec{r} = \hat{e}_x x + \hat{e}_y y$ are the transverse coordinates and $\hat{e}_x, \hat{e}_y$ are the unit vector in the x and y direction, respectively. Adopting the comoving frame coordinates, i.e., $\tau = t - z/c$ and $z = z$, and time Fourier transforming both sides of Eq. (1) yields the nonparaxial wave equation in the temporal-frequency domain,

$$\left[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2ik(\omega)\frac{\partial}{\partial z}\right]\psi(\vec{r},z,\omega) = 0, \tag{2}$$

where $k = \omega/c$ is the wave number, $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, and the time Fourier transform of the electric field is represented as

$$E(\vec{r},z,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\psi(\vec{r},z,\omega)\exp(-i\omega\tau)d\omega. \tag{3}$$

By invoking the paraxial approximation in the temporal-frequency domain, i.e.,

$$\left|\frac{\partial}{\partial z}\psi(\vec{r},z,\omega)\right| \ll |k(\omega)\psi(\vec{r},z,\omega)|,$$
$$\left|\frac{\partial^2}{\partial z^2}\psi(\vec{r},z,\omega)\right| \ll \left|k(\omega)\frac{\partial}{\partial z}\psi(\vec{r},z,\omega)\right|, \tag{4}$$

which indicates that the paraxial approximation is satisfied for each frequency (ω) component, the paraxial propagation equation in the temporal-frequency domain reads

$$\left[\nabla_\perp^2 + 2ik(\omega)\frac{\partial}{\partial z}\right]\psi(\vec{r},z,\omega) = 0. \tag{5}$$

Analogous to the derivation of the Fresnel diffraction integral, one can solve Eq. (5) in the temporal-frequency domain and an integral solution for the transverse components can be derived, i.e.,

$$\psi(\vec{r},z,\omega) = \frac{-ik}{2\pi z}\int_{-\infty}^{+\infty}\psi(\vec{r}\,',0,\omega)\exp\left[\frac{ik}{2z}(\vec{r}-\vec{r}\,')^2\right]d^2\vec{r}\,'. \tag{6}$$

Inversely time Fourier transforming both sides of Eq. (6), the integral solution for the ultrashort pulsed beam propagation in free space yields

$$E(\vec{r},z,\tau) = \frac{1}{\sqrt{2\pi}zc}\int_{-\infty}^{+\infty}\frac{\partial}{\partial\tau'}E(\vec{r}\,',0,\tau')d^2\vec{r}\,', \tag{7}$$

where $\tau' = \tau - (\vec{r}-\vec{r}\,')^2/2cz$ is the reduced time and

$$E(\vec{r}\,',0,\tau') = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{E}(\vec{r}\,',\omega,0)\exp(-i\omega\tau')d\omega \tag{8}$$

is the initial $(z=0)$ pulsed beam distribution. From Eq. (7), the pulsed-beam solution can be derived for the different initial condition and it can be found that the spatiotemporal coupling occurs generally, even though the spatial and temporal variables might be separable initially. However, the analytical solution for the paraxial beam can only be derived for some specific cases, such as Gaussian beam, Hermite-Gaussian beam, etc. Suppose $\psi(\vec{r},\omega,z)$ is a Gaussian-like beam, which satisfies Eqs. (5) and (6), i.e.,

$$\psi(\vec{r},z,\omega) = -\frac{iz_R}{q}\exp\left(ik\frac{\vec{r}^{\,2}}{2q}\right)\Phi(\vec{r},q,\omega)P(\omega), \tag{9}$$

where $q(\omega) = z - iz_R(\omega)$ is the q parameter of the Gaussian-like beam, w_0 is the waist radius, $z_R = k(\omega)w_0^2/2$ is the Rayleigh range, and $P(\omega)$ is the complex representation of the initial on-axis spectral distribution of the pulse, and $\Phi(\vec{x},q,\omega)$ is related to the order of transverse mode for the Gaussian-like beam. For an m, nth-order Hermite Gaussian beam,

$$\Phi(\vec{r},q,\omega)=\left(-\frac{q}{q^*}\right)^{(m+n)/2}H_m\left(\sqrt{\frac{z_Rk}{qq^*}}x\right)\times H_n\left(\sqrt{\frac{z_Rk}{qq^*}}y\right),\qquad(10)$$

where "*" stands for complex conjugate and for Gaussian beam, $\Phi(\vec{r},q,\omega)=1$. It is, however, very difficult to derive the analytical solution if z_R is ω-dependent. Nevertheless, since it is possible to control z_R and make it ω-independent experimentally, $q(\omega)$ can be rendered independent of ω, then

$$E(\vec{r},\tau,z)=\frac{-iz_R}{q\sqrt{2\pi}}\Phi(\vec{r},q,\tau')*P(\tau'),\qquad(11)$$

where "*" denotes the Fourier convolution and

$$\Phi(\vec{r},q,\tau')=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Phi(\vec{r},q,\omega)\exp(-i\omega\tau')d\omega,\qquad(12)$$

and

$$P(\tau')=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}P(\omega)\exp\left[i\omega\left(\frac{\vec{r}^2}{2qc}-\tau\right)\right]d\omega\qquad(13)$$

is the complex representation of the pulse. In deriving $P(\tau')$, the CAS theory [1,10–12] should be applied to avoid the spatial singularity due to SVEA. Here, $P(\omega)=2p(\omega)\theta(\omega)$,

$$p(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}p(t)\exp(-i\omega t)dt,$$

$$\theta(\omega)=\begin{cases}1, & \omega>0\\ 0, & \omega\leqslant 0\end{cases}$$

is the Heaviside step function, $p(t)=A(t)\cos(\omega_0t+\varphi)$ denotes the real representation of the initial on-axis pulse, $A(t)$ is a real function and the phase φ is independent of time t for simplicity, and ω_0 is the carrier frequency. Though the higher-order correction to the SVEA solution [23] can be applied to obtain $P(\tau')$ without using the CAS theory, it is still preferable to use the CAS representation of $P(\tau')$ to overcome the spatial singularity. For a Gaussian beam, then the pulsed Gaussian beam can be obtained,

$$E(\vec{x},\tau,z)=\frac{-iz_R}{q}P(\tau'),\qquad(14)$$

which coincides with that derived in several previous papers [10–13]. Therefore, Eq. (11) gives rise to a family of pulsed Gaussian-like beam solutions with different initial pulse distribution (different P's).

III. NONPARAXIAL CORRECTION TO THE PULSED BEAM SOLUTION

The nonparaxial correction to the paraxial pulsed beam solution given in Eqs. (7) and (11) should be based on Eq. (2), which is derived without the paraxial approximation. First, the spatial Fourier transform of the wave is

$$\tilde{\psi}(\vec{f},z,\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\psi(\vec{r},z,\omega)\exp(-2\pi i\vec{f}\cdot\vec{r})d\vec{f},\qquad(15)$$

where $\vec{f}=f_x\hat{e}_x+f_y\hat{e}_y$ is the spatial-frequency vector. Then substituting Eq. (15) into Eq. (2) yields

$$(\partial_z^2+2ik\partial_z-4\pi^2\vec{f}^2)\tilde{\psi}(\vec{f},z,\omega)=0,\qquad(16)$$

which can be solved analytically. It is well known that the electromagnetic field is an evanescent wave when $\lambda^2\vec{f}^2\geqslant 1$. Since the evanescent wave components propagate perpendicularly to and decay exponentially along the z axis, their influence can be ignored provided that the distance between the investigated transverse plane and the original transverse plane is more than several wavelengths. We will ignore the influence of the evanescent wave in the following. Therefore, the solution of Eq. (16) can be derived, i.e.,

$$\tilde{\psi}(\vec{f},z,\omega)=\exp[ikz(\sqrt{1-\lambda^2\vec{f}^2}-1)]\tilde{\psi}(\vec{f},0,\omega).\qquad(17)$$

Spatially Fourier transforming both sides of Eq. (5) yields the paraxial pulsed beam solution in the spatial-temporal-frequency domain, i.e.,

$$\tilde{\psi}^{(0)}(\vec{f},z,\omega)=\tilde{\psi}(\vec{f},0,\omega)\exp(-i\pi\lambda\vec{f}^2z).\qquad(18)$$

It is evident that Eq. (18) is the zeroth-order term of Eq. (17). Also, Eq. (18) gives Eq. (6) after the inverse spatial Fourier transform. Since $(1-x)^{1/2}=1-\frac{1}{2}x+\Sigma_{n=2}^{\infty}[(2n-3)!!/(2n)!!]x^n$, and $\exp(x)=\Sigma_{m=0}^{\infty}(x^m/m!)$, and considering Eq. (18), Eq. (17) can be expanded as

$$\tilde{\psi}(\vec{f},z,\omega)=\prod_{n=2}^{+\infty}\sum_{m=0}^{+\infty}\frac{1}{m!}\left[-ikz\frac{(2n-3)!!}{(2n)!!}\times(\lambda^2\vec{f}^2)^n\right]^m\times\tilde{\psi}^{(0)}(\vec{f},z,\omega).\qquad(19)$$

Then substituting Eq. (19) into Eq. (15), the ultrashort pulsed beam solution in the spatial-temporal-frequency domain yields

$$\psi(\vec{r},z,\omega)=\prod_{n=2}^{+\infty}\hat{T}_n\psi^{(0)}(\vec{r},z,\omega),\qquad(20)$$

where

$$\hat{T}_n=\sum_{m=0}^{+\infty}\frac{(-1)^{mn}}{m!k^{2mn}}\left[-ikz\frac{(2n-3)!!}{(2n)!!}\right]^m\nabla_{\perp}^{2mn}$$

is a ω-dependent operator. Further, substituting Eq. (20) into Eq. (3), one yields the exact solution of the ultrashort pulsed beam,

$$E(\vec{r},z,\tau)=\hat{D}_{mn}\left[\int_{-\infty}^{+\infty}\psi^{(0)}(\vec{r},z,\omega)\frac{\exp(-i\omega\tau)}{k^{m(2n-1)}}d\omega\right], \tag{21}$$

where

$$\hat{D}_{mn}=\prod_{n=2}^{+\infty}\sum_{m=0}^{+\infty}\frac{(-1)^{mn}}{m!}\left[-iz\frac{(2n-3)!!}{(2n)!!}\right]^m\nabla_\perp^{2mn}$$

is a ω-independent operator. So far, in principle, according to Eqs. (20) and (21), one can obtain the exact solution for the ultrashort pulsed beam propagation in free space, which is a correction of the paraxial solution $\psi^{(0)}$ in terms of $\hat{D}_{mn}$.

Now consider the correction to the pulsed Gaussian-like beams. For the ultrashort laser pulses emitted by solid-state mode-locked lasers, z_R is independent of ω, hence

$$E(\vec{r},z,t)=\frac{-iz_R}{q\sqrt{2\pi}}D_{mn}[\Phi_{mn}(\vec{r},q,\tau')*P(\tau')], \tag{22}$$

where "*" stands for the Fourier convolution and

$$\Phi(\vec{x},q,\mu)_{mn}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}k^{m(1-2n)}\Phi(\vec{x},q,\omega)\exp(-i\omega\mu)d\omega,$$

and $P(\tau')$ is the complex representation of the pulse, which has been defined in Eq. (13).

For a pulsed Gaussian beam, $\Phi(\vec{x},q,\omega)=1$, then

$$\Phi(\vec{x},q,\mu)_{mn}=\begin{cases}\delta(\mu), & m=0\\ -i\sqrt{\dfrac{\pi}{2}}\dfrac{(-i\mu)^{2mn-m-1}}{(2mn-m-1)!}\mathrm{sgn}(\mu) & \text{otherwise,}\end{cases} \tag{23}$$

where

$$\mathrm{sgn}(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0.\end{cases}$$

Then substituting Eq. (23) into Eq. (22), the nonparaxial pulsed Gaussian beam solution can be derived for a definite $P(\tau')$.

Next we give the magnitude analysis for the (m,n)th-order terms in the correction. Denoting $f_{max}=|\vec{f}|_{max}$ as the maximum spatial frequency, it is evident that $\lambda^2 f_{max}^2<1$. According to Eq. (19), the magnitude function is defined as

$$F_{m,n}=\frac{1}{m!}\left[-ikz\frac{(2n-3)!!}{(2n)!!}(\lambda^2 f_{max}^2)^n\right]^m. \tag{24}$$

Since $1/m!<1$ and $(2n-3)!!/(2n)!!<1$, the following relations hold for the same z:

$$|F_{m,n}|<|F_{m-1,n}|,\quad |F_{m,n}|<|F_{m,n-1}|. \tag{25}$$

Hence, when $m=M$ and $n=N$ are sufficiently large, Eq. (19) can be approximated by

$$\tilde{\psi}(\vec{f},z,\omega)=\prod_{n=2}^{N}\sum_{m=0}^{M}\frac{1}{m!}\left[-ikz\frac{(2n-3)!!}{(2n)!!}\times(\lambda^2\vec{f}^2)^n\right]^m \times\tilde{\psi}^{(0)}(\vec{f},z,\omega), \tag{26}$$

and the difference between Eqs. (26) and (19) can be ignored when the condition $F_{M,N}\ll 1$ is satisfied. Hence the nonparaxial pulsed beam solution can be expressed by a correction to the paraxial solution depicted by a finite series Eqs. (20) and (21), in which

$$\hat{T}_n=\sum_{m=0}^{M}\frac{(-1)^{mn}}{m!k^{2mn}}\left[-ikz\frac{(2n-3)!!}{(2n)!!}\right]^m\nabla_\perp^{2mn},$$

where $n=2,\ldots,N$, and

$$\hat{D}_{mn}=\prod_{n=2}^{N}\sum_{m=0}^{M}\frac{(-1)^{mn}}{m!}\left[-iz\frac{(2n-3)!!}{(2n)!!}\right]^m\nabla_\perp^{2mn}.$$

IV. DISCUSSION AND CONCLUSION

In summary, the comoving frame coordinates and the Fourier transform for time variables have been applied to obtain the nonparaxial pulsed beam propagation equation in the temporal-frequency domain. Then the paraxial equation can be derived in the temporal-frequency domain, which has the same form as that of beam propagation [1–3]. The integral solution for the ultrashort pulsed beam propagation was derived, which generally represents the propagation of an ultrashort pulsed beam in free space. As an example, the pulsed Gaussian-like beam solution was given when z_R is independent of ω, in which the complex representation of the initial on-axis pulse $P(\tau')$ can be given by CAS theory [1,10–12] or the correction method [23], and the pulsed Gaussian beam solution can be rederived.

By making use of the Fourier transform for spatial variables $\vec{r}$, and starting from the nonparaxial pulsed beam propagation equation in the temporal-frequency domain, the solution for nonparaxial pulsed beam propagation is derived as a correction to the paraxial solution, which is an infinite series containing the derivative operators $\hat{T}_{mn}$ and $\hat{D}_{mn}$. In particular, the nonparaxial pulsed Gaussian-like beam solution is given. Using magnitude analysis, it can be found that the higher-order correction terms have fewer effects than lower ones and hence the corrections can be made by a finite series with the derivative operators $\hat{T}_{mn}$ and $\hat{D}_{mn}$.

ACKNOWLEDGMENTS

This work was partially supported by the Key Project of the National Natural Science Foundation of China (under Grant No. 69789801), the Team Project of the Natural Science Foundation of Guangdong Province (under Grant No.

20003061), the Foundation of National Hi-Tech Inertial Confinement Fusion Committee, the Fok Yin Tung High Education Foundation (No. 71058), and the Foundation for the Key Young Teachers of the Ministry of Education of China. The authors' sincere thanks go to Dr. H. Pu for his careful reading and many suggestions on the manuscript.

[1] M. Born and E. Wolf, *Principles of Optics*, 7th ed. (Cambridge University Press, Cambridge, 1999).
[2] P.W. Millonni and J.H. Eberly, *Lasers* (Wiley, New York, 1988).
[3] A.E. Siegman, *Lasers* (University Science, Mill Valley, CA, 1986).
[4] I.P. Christov, Opt. Commun. **53**, 364 (1985).
[5] D. You and P.H. Bucksbaum, J. Opt. Soc. Am. B **14**, 1651 (1997).
[6] A.E. Kaplan, S.F. Straub, and P.L. Shkolnikov, Opt. Lett. **22**, 405 (1996).
[7] A.E. Kaplan, J. Opt. Soc. Am. B **15**, 951 (1998).
[8] T. Brabec and Ferenc Krausz, Phys. Rev. Lett. **78**, 3282 (1997).
[9] M.A. Porras, R. Borghi, and M. Santarsiero, Phys. Rev. E **62**, 5729 (2000).
[10] M.A. Porras, Phys. Rev. E **58**, 1086 (1998).
[11] M.A. Porras, J. Opt. Soc. Am. B **16**, 1468 (1999).
[12] M.A. Porras, Phys. Rev. A **60**, 5069 (1999).
[13] Z. Wang, Z. Zhang, Z. Xu, and Q. Lin, IEEE J. Quantum Electron. **33**, 566 (1997).
[14] S. Feng, Herbert G. Winful and R.W. Hellwarth, Opt. Lett. **23**, 385 (1998).
[15] S. Feng and H.G. Winful, Phys. Rev. E **61**, 862 (2000).
[16] S. Feng, H.G. Winful, and R.W. Hellwarth, Phys. Rev. E **59**, 4630 (1999).
[17] S. Feng and H.G. Winful, J. Opt. Soc. Am. A **16**, 2500 (1999).
[18] S. Hunsche, S. Feng, H.G. Winful, A. Leitenstorfer, M.C. Nuss, and E.P. Ippen, J. Opt. Soc. Am. A **16**, 2025 (1999).
[19] G.P. Agrawal, Opt. Commun. **157**, 52 (1998).
[20] G.P. Agrawal, Opt. Commun. **167**, 15 (1999).
[21] Richard W. Ziolkowski, Phys. Rev. A **39**, 2005 (1989).
[22] S. Feng and H.G. Winful, Phys. Rev. E **63**, 046602 (2001).
[23] M.A. Porras, Opt. Lett. **26**, 44 (2001).
[24] M. Lax, W.H. Louisell, and W.B. McKnight, Phys. Rev. A **11**, 1365 (1975).
[25] Q. Cao and X. Deng, J. Opt. Soc. Am. A **15**, 1144 (1998).
[26] Q. Cao, J. Opt. Soc. Am. A **16**, 2494 (1999).